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Bound Motion of Bodies and Paticles in the Rotating Systems

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The Lagrange theory of particle motion in the noninertial systems is applied to the Foucault pendulum, isosceles triangle pendulum and the general triangle pendulum swinging on the rotating Earth. As an analogue, planet orbiting in the rotating galaxy is considered as the giant galactic gyroscope. The Lorentz equation and the Bargmann-Michel-Telegdi equations are generalized for the rotation system. The knowledge of these equations is inevitable for the construction of LHC where each orbital proton "feels" the Coriolis force caused by the rotation of the Earth.

KEY WORDS: Foucault pendulum; triangle pendulum; gyroscope; rotating galaxy; Lorentz equation; Bargmann-Michel-Telegdi equation.

1. INTRODUCTION

In order to reveal the specific characteristics of the mechanical systems in the rotating framework, it is necessary to derive the differential equations describing the mechanical systems in the noninertial systems. We follow the text of Landau *et al.* (Landau *et al.*, 1965).

Let be the Lagrange function of a point particle in the inertial system as follows:

$$L_0 = \frac{m\mathbf{v}_0^2}{2} - U \tag{1}$$

with the following equation of motion

$$m\frac{d\mathbf{v}_0}{dt} = -\frac{\partial U}{\partial \mathbf{r}}, \tag{2}$$

where the quantities with index 0 correspond to the inertial system.

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The Lagrange equations in the noninertial system is of the same form as that in the inertial one, or,

$$\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}} = \frac{\partial L}{\partial \mathbf{r}}. \tag{3}$$

However, the Lagrange function in the noninertial system is not the same as in Eq. (1) because it is transformed.

Let us first consider the system K' moving relatively to the system K with the velocity $\mathbf{V}(t)$. If we denote the velocity of a particle with regard to system K' as $\mathbf{v}'$, then evidently

$$\mathbf{v}_0 = \mathbf{v}' + \mathbf{V}(t). \tag{4}$$

After insertion of Eq. (4) into Eq. (1), we get

$$L_0' = \frac{m\mathbf{v}'^2}{2} + m\mathbf{v}'\mathbf{V} + \frac{m}{2}\mathbf{V}^2 - U. \tag{5}$$

The function $\mathbf{V}^2$ is the function of time only and it can be expressed as the total derivation of time of some new function. It means that the term with the total derivation in the Lagrange function can be removed from the Lagrangian. We also have:

$$m\mathbf{v}'\mathbf{V}(t) = m\mathbf{V}\frac{d\mathbf{r}'}{dt} = \frac{d}{dt}(m\mathbf{r}'\mathbf{V}(t)) - m\mathbf{r}'\frac{d\mathbf{V}}{dt}. \tag{6}$$

After inserting the last formula into the Lagrange function and after removing the total time derivation we get

$$L' = \frac{mv'^2}{2} - m\mathbf{W}(t)\mathbf{r}' - U, \tag{7}$$

where $\mathbf{W} = d\mathbf{V}/dt$ is the acceleration the system K'.

The Lagrange equations following from the Lagrangian (7) are as follows:

$$m\frac{d\mathbf{v}'}{dt} = -\frac{\partial U}{\partial \mathbf{r}'} - m\mathbf{W}(t). \tag{8}$$

We see that after acceleration of the system K' the new force $m\mathbf{W}(t)$ appears. This force is fictitious one because it is not generated by the internal properties of some body.

In case that the system K' rotates with the angle velocity $\mathbf{\Omega}$ with regard to the system K, vectors $\mathbf{v}$ and $\mathbf{v}'$ are related as (Landau *et al.*, 1965)

$$\mathbf{v}' = \mathbf{v} + \mathbf{\Omega} \times \mathbf{r}. \tag{9}$$

The Lagrange function for this situation is (Landau *et al.*, 1965)

$$L = \frac{mv^2}{2} - m\mathbf{W}(t)\mathbf{r} - U + m\mathbf{v}\cdot(\mathbf{\Omega}\times\mathbf{r}) + \frac{m}{2}(\mathbf{\Omega}\times\mathbf{r})^2. \tag{10}$$

The corresponding Lagrange equations for the last Lagrange function are as follows (Landau *et al.*, 1965):

$$m\frac{d\mathbf{v}}{dt} = -\frac{\partial U}{\partial \mathbf{r}} - m\mathbf{W} + m\mathbf{r}\times\dot{\boldsymbol{\Omega}} + 2m\mathbf{v}\times\boldsymbol{\Omega} + m\boldsymbol{\Omega}\times(\mathbf{r}\times\boldsymbol{\Omega})\,. \tag{11}$$

We observe in Eq. (11) three so called inertial forces. The force $m\mathbf{r}\times\dot{\boldsymbol{\Omega}}$ is connected with the nonuniform rotation of the system K' and the forces $2m\mathbf{v}\times\boldsymbol{\Omega}$ and $m\boldsymbol{\Omega}\times\mathbf{r}\times\boldsymbol{\Omega}$ correspond to the uniform rotation. The force $2m\mathbf{v}\times\boldsymbol{\Omega}$ is so called the Coriolis force and it depends on the velocity of a particle. The force $m\boldsymbol{\Omega}\times\mathbf{r}\times\boldsymbol{\Omega}$ is called the centrifugal force. It is perpendicular to the rotation axes and the magnitude of it is $m\varrho\omega^2$, where ϱ is the distance of the particle from the rotation axis.

Equation (11) can be applied to many special cases. We apply it first to the case of the mathematical pendulum swinging in the gravitational field of the rotating Earth. In other words, to the so called Foucault pendulum.

2. FOUCAULT PENDULUM

Foucault pendulum was studied by Léon Foucault (1819–1868) as the big mathematical pendulum with big mass m swinging in the gravitational field of the Earth. He used a 67 m long pendulum in the Panthéon in Paris and showed the astonished public that the direction of its swing changed over time rotating slowly. The experiment proved that the earth rotates. If the earth would not rotate, the swing would always continue in the same direction.[2]

If we consider the motion in the system only with uniform rotation, then we write equation (11) in the form:

$$m\frac{d\mathbf{v}}{dt} = -\frac{\partial U}{\partial \mathbf{r}} + 2m\mathbf{v}\times\boldsymbol{\Omega} + m\boldsymbol{\Omega}\times\mathbf{r}\times\boldsymbol{\Omega}. \tag{12}$$

In case of the big pendulum, the vertical motion can be neglected and at the same time the term with Ω^2. The motion of this pendulum is performed in the horizontal plane xy. The corresponding equations are as follows (Landau *et al.*, 1965):

$$\ddot{x} + \omega^2 x = 2\Omega_z\dot{y}, \quad \ddot{y} + \omega^2 y = -2\Omega_z\dot{x}, \tag{13}$$

where ω is the frequency of the mathematical pendulum without rotation of the Earth, or $\omega = 2\pi/T$ and (Landau *et al.*, 1965): $T \approx 2\pi\sqrt{l/g}$, where T is the period of the pendulum oscillations, l is the length of the pendulum and g is the Earth acceleration.

[2] Author performed the experiment with the Foucault pendulum inside of the rotunda of the Flower garden in Kroměříž (Moravia, Czech Republic).

After multiplication of the second equation of (13) by the imaginary number i and summation with the first equation, we get:

$$\ddot{\xi} + 2i\Omega_z\dot{\xi} + \omega^2\xi = 0 \quad (14)$$

for the complex quantity $\xi = x + iy$. For the small angle rotation frequency Ω_z of the Earth with regard to the oscillation frequency ω, $\Omega_z << \omega$, we easily find the solution in the form:

$$\xi = e^{-i\Omega_z t}(A_1 e^{i\omega t} + A_2 e^{-i\omega t}), \quad (15)$$

or,

$$x + iy = e^{-i\Omega_z t}(x_0 + iy_0), \quad (16)$$

where functions $x_0(t)$, $y_0(t)$ are the parametric expression of the motion of the pendulum without the Earth rotation. If the complex number is expressed in the trigonometric form of (16), the Ω_z is the rotation of the complex number $x_0 + iy_0$. The physical meaning of Eq. (16) is, that the plane of the Foucault pendulum rotates with the frequency Ω_z with regard to the Earth.

3. THE TRIANGLE PENDULUM

The triangle pendulum is the analogue of the Foucault pendulum with the difference that the pendulum is a rigid system composed from two rods forming the triangle ABC. In the isosceles triangle it is $AC = CB = l =$ const. The legs $AC = CB$ are supposed to be prepared from the nonmetal and nonmagnetic material, with no interaction with the magnetic field of the Earth. Point C is a vertex at which the pendulum is hanged. The vertex is realized by the very small ball. Points A and B are not connected by the rod. The angle $ACB = \alpha$.

To be pedagogical clear, let us give first the known theory of the simple mathematical pendulum (Amelkin, 1987).

The energetical equation of the pendulum is of the form (φ is the deflection angle from vertical, φ_0 is the initial deflection angle from vertical):

$$\frac{mv^2}{2} - mgl\cos\varphi = -mgl\cos\varphi_0, \quad (17)$$

from which follows, in the polar coordinates with $v = l\dot{\varphi}$

$$\ddot{\varphi} + \frac{g}{l}\sin\varphi = 0. \quad (18)$$

We have for the very small angle φ that $\sin\varphi \approx \varphi$, $x \approx l\varphi$ and it means that from the last equation follows the equation for the harmonic oscillator

$$\ddot{x} + \frac{g}{l}x = 0. \quad (19)$$

The rigorous derivation of the period of pendulum follows from Eq. (17). With $v = ds/dt = ld\varphi/dt$, we get

$$\frac{l}{2}\left(\frac{d\varphi}{dt}\right)^2 = g(\cos\varphi - \cos\varphi_0). \tag{20}$$

Then,

$$dt = \sqrt{\frac{l}{2g}}\frac{d\varphi}{\sqrt{\cos\varphi - \cos\varphi_0}}. \tag{21}$$

For the period T of the pendulum, we have from the last formula:

$$\frac{T}{4} = \sqrt{\frac{l}{2g}}\int_0^{\varphi_0}\frac{d\varphi}{\sqrt{\cos\varphi - \cos\varphi_0}}. \tag{22}$$

Using relations $\cos\varphi = 1 - 2\sin^2\varphi/2$, $\cos\varphi_0 = 1 - 2\sin^2\varphi_0/2$, and substitution $\sin\varphi/2 = k\sin\chi$, with $k = \sin\varphi_0/2$, we get

$$d\varphi = \frac{2\sqrt{k^2 - \sin^2\chi/2}}{\sqrt{1 - k^2\sin^2\chi}}d\chi \tag{23}$$

and finally

$$T = 4\sqrt{\frac{l}{g}}\int_0^{\pi/2}\frac{d\chi}{\sqrt{1 - k^2\sin^2\chi}}, \tag{24}$$

where the integral in the last formula is so called the elliptic integral, which cannot be evaluated explicitly but only in the form of series.

Now, let us go back to the isosceles triangle pendulum, which differs from the mathematical pendulum in such a way that it is a rotating system. We write in the polar coordinates instead of the equation (18) the equation for the rotating system which ia called the physical pendulum:

$$J\ddot{\varphi} = -(\Sigma m_i)ga\sin\varphi, \tag{25}$$

where J is the moment of inertia of the triangle pendulum with the determination

$$J = \Sigma m_i l_i^2 = 2ml^2 \tag{26}$$

and a is the distance of the center-of-mass to the axis of rotation, or,

$$a = l\cos(\alpha/2). \tag{27}$$

It is easy to see the equation of motion is

$$\ddot{\varphi} + (g/l)\cos(\alpha/2)\sin\varphi = 0, \tag{28}$$

which has the limiting form

$$\ddot{\varphi} + (g/l)\cos(\alpha/2)\varphi = 0, \tag{29}$$

for the small deflection angles and it means that the frequency of oscillations is

$$\omega = \sqrt{\frac{g}{l}\cos(\alpha/2)}. \tag{30}$$

For $\alpha = 0$ we get the frequency of the mathematical pendulum. It is evident that the triangle pendulum behaves on the rotating Earth as the Foucault pendulum and it can be used as the table pendolino experiment for the demonstration of the Earth rotation.

The triangle pendulum with equal sides can be generalized to the situation with $AC = l_1$, $BC = l_2$ and with masses m_1, m_2. Then, it the equation of motion of such generalized triangle pendulum is equation (25) with $J = m_1 l_1^2 + m_2 l_2^2$ and with a being given by the cosine theorem in the triangle ABC (the angle CAB = α_1, the angle ABC = α_2)

$$a^2 = l_1^2 + x_1^2 - 2l_1x_1\cos\alpha_1 = l_2^2 + x_2^2 - 2l_2x_2\cos\alpha_2, \tag{31}$$

where x_1, x_2 can be determined from equations

$$x_1 + x_2 = AB = \sqrt{l_1^2 + l_2^2 - 2l_1l_2\cos\alpha}, \quad x_1/x_2 = m_2/m_1 \tag{32}$$

The last equation means that the components x_1, x_2 with $x_1 + x_2 = AB$ determine the position of the center-of-mass T, which evidently lies on the line AB between points A and B CT $= \alpha$.

For $l_1 = l_2 = l$ and $m_1 = m_2 = m$, we get the isosceles pendulum and for $\alpha = 0$, we get the original simple mathematical pendulum.

The mathematical and physical analysis of the general triangle pendulum shows us that this pendulum has the same behavior as the Foucault pendulum. Or, in other words we can denote it as the triangle Foucault pendulum.

4. THE GALACTIC GYROSCOPE

The gyroscope is usually defined as a device for measuring or maintaining orientation based on the principle of conservation of angular momentum. The essence of the device is the spinning wheel. We will show that the planet orbiting in the rotating galaxy is the galactic gyroscope because the orientation of the orbit is conserved reminding the classical gyroscope.

The force acting on the planet with mass m is according to Newton law

$$F = -G\frac{mM}{r^2}, \tag{33}$$

where M is the mass of Sun, r being the distance from m to the Sun.

The corresponding equations of motion in the coordinate system x and y are as follows

$$m\ddot{x} = -G\frac{mM}{r^2}\cos\varphi; \quad m\ddot{y} = -G\frac{mM}{r^2}\sin\varphi, \tag{34}$$

or, with $\sin\varphi = y/r, \cos\varphi = x/r$,

$$\ddot{x} = -\frac{kx}{r^3}; \quad \ddot{y} = -\frac{ky}{r^3}, \quad k = GM, \quad r = \sqrt{x^2 + y^2} \tag{35}$$

Using $x = r\cos\varphi$, $y = r\sin\varphi$, we get instead of equations (35):

$$(\ddot{r} - r\dot{\varphi}^2)\cos\varphi - (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\sin\varphi = -\frac{k\cos\varphi}{r^2} \tag{36}$$

$$(\ddot{r} - r\dot{\varphi}^2)\sin\varphi + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\cos\varphi = -\frac{k\sin\varphi}{r^2}. \tag{37}$$

In case that the motion of the planet is performed in the rotation system of a galaxy the equations (36), (37) are written in the form ($\Omega_z = \Omega$)

$$(\ddot{r} - r\dot{\varphi}^2)\cos\varphi - (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\sin\varphi = -\frac{k\cos\varphi}{r^2} + 2\Omega\dot{y} \tag{38}$$

$$(\ddot{r} - r\dot{\varphi}^2)\sin\varphi + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\cos\varphi = -\frac{k\sin\varphi}{r^2} - 2\Omega\dot{x}, \tag{39}$$

or,

$$(\ddot{r} - r\dot{\varphi}^2)\cos\varphi - (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\sin\varphi = -\frac{k\varphi}{r^2} + 2\Omega(\dot{r}\sin\varphi + r\cos\varphi\dot{\varphi}) \tag{40}$$

$$(\ddot{r} - r\dot{\varphi}^2)\sin\varphi + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\cos\varphi = -\frac{k\sin\varphi}{r^2} - 2\Omega(\dot{r}\cos\varphi - r\sin\varphi\dot{\varphi}). \tag{41}$$

After multiplication of Eq. (40) by $\sin\varphi$ and Eq. (41) by $\cos\varphi$ and after their subtraction we get

$$2\dot{r}\dot{\varphi} + r\ddot{\varphi} = -2\Omega\dot{r}, \tag{42}$$

or,

$$\frac{d}{dt}(r^2\dot{\varphi}) = -\Omega\frac{d}{dt}(r^2), \tag{43}$$

or,

$$\dot{\varphi} = -\Omega. \tag{44}$$

It means that the angle velocity of the ellipse of a planet inside the rotating galaxy is $-\Omega$ which is the opposite angle velocity of the galaxy with regard to vacuum of universe (Of course that the additional fundametal solution of Eq. (42) is $\dot{\varphi} = \text{const}/r^2$).

Let us only remark that here we consider the well defined galaxy as the galaxy of elliptical form and not of the chaotic form. We do not consider here the " galaxy rotation problem"—the discrepancy between the observed rotation speeds of matter in the disk portion of spiral galaxies and the predictions of Newton dynamics considering the luminous mass—which is for instance discussed in *http://en.wikipedia.org/wiki/Galaxy_spiral_problem.*

5. ROTATING LHC FROM THE VIEWPOINT OF GRG

Now, the question arises what is the description of the rotation in the general theory of relativity. If we use the the Minkowski element

$$ds^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 \tag{45}$$

and the nonrelativistic transformation to the rotation system (Landau *et al.*, 1988)

$$x' = x \cos \Omega t - y \sin \Omega t, \quad y' = x \sin \Omega t + y \cos \Omega t, \quad z = z' \tag{46}$$

then we get:

$$ds^2 = [c^2 - \Omega^2(x^2 + y^2)]dt^2 - dx^2 - dy^2 - dz^2 + 2\Omega y\, dx\, dt - 2\Omega x\, dy\, dt, \tag{47}$$

which is not relativistically invariant.

Frequently, the modified notation is used in the literature for the description of the metric on the rotation Earth (Grib *et al.*, 1987). The following tetrade system connected with the observer is chosen. The unite vector $\mathbf{e}_z$ lies on the prime going from the center of rotation of the Earth to the place of the observer on the Earth. The vector $\mathbf{e}_y$ is oriented to the North pole and lies on the meridian, the vector $\mathbf{e}_x$ is perpendicular to $\mathbf{e}_z$ and $\mathbf{e}_y$ and it lies in the direction of the Earth rotation. The angle velocity of basic vector is identical with the angle velocity of the Earth. The acceleration $\mathbf{a}$ in the observer system is the sum of the gravitational acceleration $\mathbf{g}$ and the centrifugal acceleration $\mathbf{a}_c$. The metrics in a such system is given by the appropriate components of the following line element:

$$ds^2 = \left(1 + \frac{2z}{c^2}(\Omega^2 R \cos^2 \alpha - g)\right)(dct)^2 - 2z\Omega(dct)dx - dx^2, \tag{48}$$

where R is the radius of the Earth, Ω is the angle velocity of the Earth rotation, g is the gravitational acceleration in the place of experiment, α is the Earth width of the experimental arrangement.

The metrical tensor following from the line element (48) in the first approximation is evidently given by the relations (Grib *et al.*, 1987):

$$g^{00} = +\frac{2zg}{c^2}, \quad g^{01} = g^{10} = -\frac{\Omega}{c} z \cos\alpha, \quad g^{11} = -1. \tag{49}$$

The correctness of the transformation between inertial and rotation system is necessary because it enables to describe the motion of the particle and spin in the LHC by the general relativistic methods. The basic idea is the generalization of the so called Lorentz equation for the charged particle in the electromagnetic field $F^{\mu\nu}$ (Landau *et al.*, 1988):

$$mc\frac{dv^\mu}{ds} = \frac{e}{c}F^{\mu\nu}v_\nu. \tag{50}$$

In other words, the normal derivative must be replaced by the covariant one and we get the general relativistic equation for the motion of a charged particle in the electromagnetic field and gravity (Landau *et al.*, 1988):

$$mc\left(\frac{dv^\mu}{ds} + \Gamma^\mu_{\alpha\beta}v^\alpha v^\beta\right) = \frac{e}{c}F^{\mu\nu}v_\nu, \tag{51}$$

where

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\lambda}\left(\frac{\partial g_{\lambda\alpha}}{\partial x^\beta} + \frac{\partial g_{\lambda\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\lambda}\right) \tag{52}$$

are the Christoffel symbols derived in the Riemann geometry theory (Landau *et al.*, 1988).

In case that we consider motion in the rotating system, then it is necessary to insert the metrical tensor $g_{\mu\nu}$, following from the Minkowski element for the rotation system. The construction of LHC with orbiting protons must be in harmony with equation (51) because orbital protons "feels" the Coriolis force from the rotation of the Earth.

The analogical situation occurs for the motion of the spin. While the original Bargmann-Michel-Telegdi equation for the spin motion is as follows (Berestetzkii *et al.*, 1988)

$$\frac{da^\mu}{ds} = 2\mu F^{\mu\nu}a_\nu - 2\mu'\mu v^\mu F^{\alpha\beta}v_\alpha a_\beta, \tag{53}$$

where $\mu' = \mu - e/2m$ and a_μ is the axial vector, which follows also from the classical limit of the Dirac equation as $\bar{\psi}i\gamma_5\gamma_\mu\psi$ (Rafanelli *et al.*, 1964; Pardy, 1973), the general relativistic generalization of the Bargmann-Michel-Telegdi equation can be obtained by the analogical procedure which was performed with the Lorentz equation. Or,

$$\left(\frac{da^\mu}{ds} + \Gamma^\mu_{\alpha\beta}v^\alpha a^\beta\right) = 2\mu F^{\mu\nu}a_\nu - 2\mu'\mu v^\mu F^{\alpha\beta}v_\alpha a_\beta, \tag{54}$$

where in case of the rotating system the metrical tensor $g_{\mu\nu}$ must be replaced by the metrical tensor of the rotating system. Then, the last equation will describe the motion of the spin in the rotating system.

The motion of the polarized proton in LHC will be described by the last equation because our Earth rotates. During the derivation we wrote $\Gamma^{\mu}_{\alpha\beta} v^{\alpha} a^{\beta}$ and not $\Gamma^{\mu}_{\alpha\beta} v^{\alpha} v^{\beta}$, because every term must be the axial vector. In other words, the last equation for the motion of the spin in the rotating system was not strictly derived but created with regard to the philosophy that physics is based on the creativity and logic (Pardy, 2005).

On the other hand, the Eq. (54) must evidently follow from the Dirac equation in the rotating system, by the same WKB methods which were used by Rafanelli, Schiller and Pardy (Rafanelli and Schiller, 1964; Pardy, 1973). The derived BMT equation in the metric of the rotation of the Earth are fundamental for the proper work of LHC because every orbital proton of LHC "feels" the rotation of the Earth and every orbital proton spin "feels" the Earth rotation too. So, LHC needs equations (51) and (54) and vice versa.

6. DISCUSSION

We have presented the Lagrange theory of the noninertial classical systems and we applied the theory to the so called Foucault pendulum, the isosceles triangle pendulum with two equal masses and to the triangle pendulum with the nonequal legs and masses. We have shown that Every pendulum is suitable for the demonstration of the rotation of the earth. The isosceles triangle with two equal masses and the triangle with the nonequal legs and masses fixed to the ball swimming on the water was not considered. The article is the modified and improved version of the previous author text (Pardy, 2006).

We know from history of science that Galileo Galilei (1564–1642)—Italian scientist and philosopher—studied the mathematical pendulum before Foucault. While in a Pisa cathedral, he noticed that a chandelier was swinging with the same period as timed by his pulse, regardless of his amplitude. It is probable, that Galileo noticed the rotation of the swinging plane of the pendulum. However, he had not used this fact as the proof of the Earth rotation when he was confronted with the Inquisition tribunal. Nevertheless, his last words were "E pur si muove."

For the demonstration of the galaxy rotation, we have analyzed the elliptical motion of our planet and we have shown that the orbital motion of our planet can be used as gigantic gyroscope for the proof of the rotation of our galaxy in the universe. The orbit of our planet with regard to the rest of the universe has the stable stationary position while the galaxy rotates. The orbital planetary stability can be used as the method of the investigation of the rotation of all galaxies in the rest of the universe. To our knowledge this method was not still used in the galaxy astrophysics.

It is possible to consider also the rotation of the Universe. If we define Universe as the material bodies immersed into vacuum, then the rotation of the Universe is physically meaningful and the orbit of our planet is of the constant position with regard to the vacuum as the rest system. The idea that the vacuum is the rest system is physically meaningful because only vacuum is the origin of the inertial properties of every massive body. In other words, the inertial mass m in the Newton-Euler equation $F = ma$ is the result of the interaction of the massive body with vacuum and in no case it is the result of the Mach principle where the inertial mass is generated by the mass of rest of the Universe. At present time, everybody knows that Mach principle is absolutely invalid for all time of the existence of Universe.

The theory discussed in our article can be also applied to the pendulum where the fiber is elastic. The corresponding motion is then described by the wave equation with the initial and boundary conditions.

It is evident that there are many physical problems, classical and quantum mechanical considered in the rotation system. Some problems were solved and some problems will be solved in the future. Let us define some of these problems.

Mössbauer effect in the rotating system, Schrödinger equation for a particle in the rotating system, Schrödinger equation for the pendulum in the inertial system and in the rotating system, Schrödinger equation of H-atom in the rotating system, Schrödinger equation of harmonic oscillator in the rotating system, the Čerenkov effect in the rotating dielectric medium, the relic radiation in the rotating galaxy, the N-dimensional blackbody radiation in the rotating system, conductivity and superconductivity in the rotating system, laser pulse in the rotating system, Berry phase, Sagnac effect, and so on. All these problems can be formulated classically, or in the framework of the general theory of relativity with the Γ-connections corresponding to the geometry of the rotating system. We hope that the named problems are interesting and their solution will be integral part of the theoretical physics.

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